

浙江省 A9 协作体 返校联考

高三数学参考答案

一、单选题：本题共 8 小题，每小题 5 分，共 40 分.

题目	1	2	3	4	5	6	7	8
答案	B	D	C	A	A	C	B	C

第 8 题解答：

$\because a = \ln 10 < \ln e^3 = 3, b > 3, c > 3, \therefore a$ 最小

$\because$ 由 $y = 2^x$ 与 $y = x^2$ 的图象知，当 $2 < x < 4$ 时， $2^x < x^2$ ，

$\therefore$ 取 $x = \sqrt{10}$ ，得 $2^{\sqrt{10}} < (\sqrt{10})^2 = 10$ ，

$\therefore \sqrt{10} < \log_2 10$ ，即 $b < c$ ，因此有： $a < b < c$ ，选 C

二、多项选择题：本题共 3 小题，每小题 6 分，共 18 分. 在每小题给出的选项中，有多项符合题目要求. 全部选对的得 6 分，部分选对得部分分，有选错的得 0 分.

题目	9	10	11
答案	BCD	BCD	ABD

三、填空题：本题共 3 小题，每小题 5 分，共 15 分.

12. 20

13. 3

14. 3

第 14 题解答：

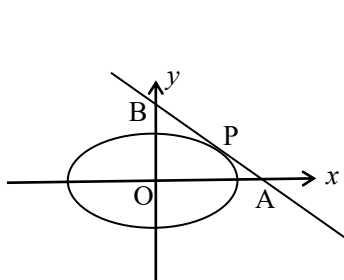
法一：由对称性知，不妨设点 $P(x_0, y_0)$ 在第一象限，则切线方程：

$$\frac{x_0 x}{9} + \frac{y_0 y}{4} = 1, \therefore A\left(\frac{9}{x_0}, 0\right), B\left(0, \frac{4}{y_0}\right), \therefore \begin{cases} (x_0 - \frac{9}{x_0})^2 + y_0^2 = 4 \\ \frac{x_0^2}{9} + \frac{y_0^2}{4} = 1 \end{cases}, \text{解得: } \begin{cases} x_0 = \frac{3\sqrt{15}}{5} \\ y_0 = \frac{2\sqrt{10}}{5} \end{cases}$$

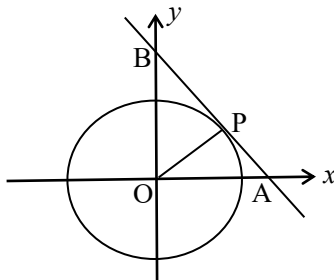
$$\therefore P\left(\frac{3\sqrt{15}}{5}, \frac{2\sqrt{10}}{5}\right), A(\sqrt{15}, 0), B(0, \sqrt{10}), \therefore |PB|^2 = 9, \text{得 } |PB| = 3.$$

法二：将椭圆仿射变换成圆： $x^2 + y^2 = r^2$ ，则过圆上一点 P 作切线 l 分别交 x 轴、 y 轴于 A 、 B 两点，由射影定理知： $|PA| \cdot |PB| = r^2$ ，故类比还原成椭圆的结论是： $|PA| \cdot |PB| = ab$ ，

$$\therefore 2|PB| = 2 \times 3, \therefore |PB| = 3.$$



法一图



法二图

四、解答题：本题共 5 小题，共 77 分.

注：阅卷时，每小题答案正确，且有过程，则为满分.若答案错误，则参考提供的分值.有较多解法的题目，则不同的解答方法可酌情对应给分.

15. 解：

$$(1) \because \angle ADC = 150^\circ, \therefore \angle ADB = 30^\circ, \because \angle B = 60^\circ, \therefore \angle BAD = 90^\circ; \\ \therefore AB = 1, AD = \sqrt{3} \quad \dots\dots\dots (4 \text{ 分})$$

$$\therefore AC^2 = 3 + 1 - 2 \times \sqrt{3} \times 1 \times \cos 150^\circ = 7, \therefore AC = \sqrt{7} \quad \dots\dots\dots (6 \text{ 分})$$

$$(2) \because BP = \frac{3}{2}, \therefore AP^2 = 1 + \left(\frac{3}{2}\right)^2 - 2 \times 1 \times \frac{3}{2} \times \cos 60^\circ = \frac{7}{4} \\ \dots\dots\dots (10 \text{ 分})$$

$$\therefore \cos \angle APC = \frac{\frac{7}{4} + \frac{9}{4} - 7}{2 \times \frac{\sqrt{7}}{2} \times \frac{3}{2}} = -\frac{2\sqrt{7}}{7} \quad \dots\dots\dots (13 \text{ 分})$$

16. 解：

$$(1) \because f(1) = e - a, \therefore \text{切点 } (1, e - a), \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore f'(x) = e^x - 2ax \quad \dots\dots\dots (4 \text{ 分})$$

$$\therefore k = f'(1) = e - 2a = 0, \text{得 } a = \frac{e}{2}, \quad \dots\dots\dots (6 \text{ 分})$$

$$\text{切线方程: } y = \frac{e}{2} \quad \dots\dots\dots (8 \text{ 分})$$

$$(2) \because f(x) = e^x - ax^2 = 0, \therefore a = \frac{e^x}{x^2} \quad \dots\dots\dots (10 \text{ 分})$$

$$\text{设 } g(x) = \frac{e^x}{x^2} \therefore g'(x) = \frac{(x-2)e^x}{x^3} \quad \dots\dots\dots (12 \text{ 分})$$

$\therefore x \in (0, 2)$ 时, $g(x)$ 为减函数; $\therefore x \in (2, +\infty)$ 时, $g(x)$ 为增函数;

$$\therefore g_{\min}(x) = g(2) = \frac{e^2}{4} \quad \dots\dots\dots (14 \text{ 分})$$

$$\therefore a > \frac{e^2}{4} \quad \dots\dots\dots (15 \text{ 分})$$

17. 证：

$$(1) \because ABCD \text{ 为矩形}, \therefore AD \perp AB \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{又 } \because C'O \perp \text{面 } ABCD, \therefore C'O \perp AD \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore AB \cap C'O = O, \therefore AD \perp \text{面 } ABC' \quad \dots\dots\dots (3 \text{ 分})$$

$$\therefore AD \perp BC' \quad \dots\dots\dots (4 \text{ 分})$$

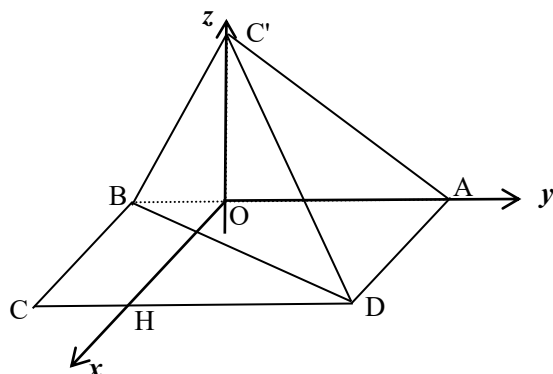
$$(2) \text{ 由 (1) 知: } AD \perp BC', \text{ 又 } \because BC \perp CD, \therefore BC' \perp C'D \quad \dots\dots\dots (6 \text{ 分})$$

$$\therefore AD \cap C'D = D, \therefore BC' \perp \text{面 } ADC' \quad \dots\dots\dots (7 \text{ 分})$$

$$\therefore BC' \subset \text{面 } DBC', \therefore \text{面 } DBC' \perp \text{面 } ADC' \quad \dots\dots\dots (8 \text{ 分})$$

解：

(3)



过点 O 作 $OH \parallel AD$, 显然 $OH \perp$ 面 ABC' . 以 O 为原点, 如图建系.

在 $Rt\triangle ABC'$ 中, $\because (BC')^2 = BO \cdot BA, \therefore 9 = BO \cdot 3\sqrt{3}, \therefore BO = \sqrt{3}, OA = 2\sqrt{3}$.

$$A(0, 2\sqrt{3}, 0), B(0, -\sqrt{3}, 0), C'(0, 0, \sqrt{6}), D(3, 2\sqrt{3}, 0) \quad \dots\dots\dots (11 \text{ 分})$$

$$\overrightarrow{AB} = (0, -3\sqrt{3}, 0), \overrightarrow{BC'} = (0, \sqrt{3}, \sqrt{6}), \overrightarrow{BD} = (3, 3\sqrt{3}, 0)$$

$$\text{求得平面 } BC'D \text{ 的法向量 } \vec{n} = (\sqrt{6}, -\sqrt{2}, 1) \quad \dots\dots\dots (13 \text{ 分})$$

$$\sin \alpha = \left| \cos \langle \overrightarrow{AB}, \vec{n} \rangle \right| = \frac{|\overrightarrow{AB} \cdot \vec{n}|}{|\overrightarrow{AB}| \cdot |\vec{n}|} = \frac{3\sqrt{6}}{3\sqrt{3} \times 3} = \frac{\sqrt{2}}{3}. \quad \dots\dots\dots (15 \text{ 分})$$

(注: 本题 (1) (2) 两问一开始建系也可以, 参考提供的分值相应给分即可)

18. 解:

$$(1) \because e = \frac{c}{a} = 3, \therefore c = 3a, \because b = 2\sqrt{2}, \therefore c^2 = a^2 + b^2 = a^2 + 8 = 9a^2, \therefore a = 1$$

$$\text{双曲线方程: } x^2 - \frac{y^2}{8} = 1 \quad \dots\dots\dots (5 \text{ 分})$$

$$(2) \text{ (i) } F_2(3, 0), k = -1, \text{ 直线 } l \text{ 的方程为: } y = -x + 3, \quad \dots\dots\dots (6 \text{ 分})$$

$$\text{设 } A(x_1, y_1), B(x_2, y_2), \text{ 则: } \begin{cases} y = -x + 3 \\ x^2 - \frac{y^2}{8} = 1 \end{cases}$$

$$\text{消元得: } 7x^2 + 6x - 17 = 0, \Delta = 36 + 4 \times 7 \times 17 = 4 \times 128, \quad \dots\dots\dots (8 \text{ 分})$$

$$\therefore |AB| = \sqrt{1+k^2} |x_1 - x_2| = \frac{32}{7} \quad \dots\dots\dots (10 \text{ 分})$$

(ii) 双曲线 C 的方程为 $x^2 - \frac{y^2}{8} = 1$, 焦点 $F_1(-3, 0), F_2(3, 0)$. 由题设知, 直线 l 的斜率必存在, 故设 $l: y = k(x - 3), A(x_1, y_1), B(x_2, y_2) (x_1 \leq -1, x_2 \geq 1)$.

$$\text{由} \begin{cases} y = k(x-3) \\ x^2 - \frac{y^2}{8} = 1 \end{cases} \text{得} (k^2 - 8)x^2 - 6k^2x + (9k^2 + 8) = 0, \text{ 所以} \begin{cases} x_1 + x_2 = \frac{6k^2}{k^2 - 8} \\ x_1 \cdot x_2 = \frac{9k^2 + 8}{k^2 - 8} \end{cases}, \dots\dots (12 \text{ 分})$$

$$\text{由} |AF_1| = |BF_1|, \text{ 所以} (x_1 + 3)^2 + k^2(x_1 - 3)^2 = (x_2 + 3)^2 + k^2(x_2 - 3)^2, \text{ 即} \\ (x_1 - x_2)[(x_1 + x_2 + 6) + k^2(x_1 - x_2 - 6)] = 0,$$

$$\text{所以} \frac{6k^2}{k^2 - 8} + 6 + k^2 \left(\frac{6k^2}{k^2 - 8} - 6 \right) = 0, \text{ 解得} k^2 = \frac{4}{5}. \dots\dots (14 \text{ 分})$$

$$\text{此时, 有} |AF_2| \cdot |BF_2| = \sqrt{1+k^2} |x_1 - 3| \cdot \sqrt{1+k^2} |x_2 - 3| = (1+k^2) |x_1 x_2 - 3(x_1 + x_2) + 9| = 16,$$

$$|AB|^2 = (x_1 - x_2)^2 + k^2(x_1 - x_2)^2 = (1+k^2)[(x_1 + x_2)^2 - 4x_1 x_2] = 16. \dots\dots (16 \text{ 分})$$

$$\text{所以} |AF_2| \cdot |BF_2| = |AB|^2, \text{ 即} |AF_2|, |AB|, |BF_2| \text{ 成等比数列}. \dots\dots (17 \text{ 分})$$

19. 解:

$$(1) p(X=2) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{5}{9}, \quad p(X=3) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{9} \dots\dots (4 \text{ 分})$$

$$(2) p(X=2k) = p^{k-1} q^{k-1} p^2 + p^{k-1} q^{k-1} q^2 = (p^2 + q^2) p^{k-1} q^{k-1} \dots\dots (7 \text{ 分})$$

$$p(X=2k+1) = p^{k-1} q^k p^2 + p^k q^{k-1} q^2 = (p+q) p^k q^k = (pq)^k \dots\dots (10 \text{ 分})$$

(3) (i) 法一:

$$\begin{aligned} E(X) &= \sum_{n=2}^{+\infty} n \cdot p(X=n) = \sum_{k=1}^{+\infty} 2k \cdot p(X=2k) + \sum_{k=1}^{+\infty} (2k+1) \cdot p(X=2k+1) \\ &= \sum_{k=1}^{+\infty} 2k \cdot (p^2 + q^2)(pq)^{k-1} + \sum_{k=1}^{+\infty} (2k+1) \cdot (pq)^k \\ &= 2(p^2 + q^2) \sum_{k=1}^{+\infty} k(pq)^{k-1} + 2pq \sum_{k=1}^{+\infty} k(pq)^{k-1} + \sum_{k=1}^{+\infty} (pq)^k \\ &= 2(p^2 + q^2) \cdot \frac{1}{(1-pq)^2} + 2pq \cdot \frac{1}{(1-pq)^2} + \frac{pq}{1-pq} \dots\dots (14 \text{ 分}) \\ &= \frac{2(p^2 + q^2) + 2pq + pq(1-pq)}{(1-pq)^2} = \frac{2(p^2 + q^2) + 3pq - (pq)^2}{(1-pq)^2} \\ &= \frac{2(p+q)^2 - pq - (pq)^2}{(1-pq)^2} = \frac{2-pq - (pq)^2}{(1-pq)^2} = \frac{(1-pq)(2+pq)}{(1-pq)^2} \\ &= \frac{2+pq}{1-pq} \end{aligned}$$

法二: 设 $p(X \geq k)$ 表示第 $k-1$ 次比赛未结束; 则有:

$$p(X \geq 1) = 1, \quad p(X \geq 2k) = q^{k-1} p^k + p^{k-1} q^k = p^{k-1} q^{k-1} (p+q) = (pq)^{k-1}, \quad p(X \geq 2k+1) = 2(pq)^k$$

$$\therefore E(X) = p(X \geq 1) + p(X \geq 2k) + p(X \geq 2k+1) = 1 + \sum_{k=1}^{+\infty} (pq)^{k-1} + \sum_{k=1}^{+\infty} 2(pq)^k \dots\dots (14 \text{ 分})$$

$$= 1 + \frac{1}{1-pq} + \frac{2pq}{1-pq} = \frac{2+pq}{1-pq}$$

$$\begin{aligned}
 \text{(ii)} \quad & \because pq \leq \left(\frac{p+q}{2}\right)^2 = \frac{1}{4}, \quad p=q=\frac{1}{2} \text{ 时取等号;} \\
 & \therefore E(X) = \frac{2+pq}{1-pq} = \frac{-(1-pq)+3}{1-pq} = \frac{3}{1-pq} - 1 \leq \frac{3}{1-\frac{1}{4}} - 1 = 3 \quad \dots\dots (17 \text{ 分})
 \end{aligned}$$